TRANSVERSE VIBRATIONAL ANALYSIS OF A SIMPLY SUPPORTED BEAM

A Thesis submitted in partial fulfillment of the requirements for the Degree of

Bachelor of Technology

IN

Mechanical Engineering

By

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CERTIFICATE

This is to certify that the thesis entitled, **"Transverse vibrational Analysis of simply supported beam"** submitted by **ANKIT SINGH** in partial fulfillment of the requirement for the award of Bachelor of Technology degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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CONTENTS

ABSTRACT

The bending phenomenon is common in simply supported beams as the beams are subjected to flexural loading in design considerations. In this paper, the effect of free vibration of the hinged beam was investigated using a finite element method and the basic understanding of the influence of applied force on natural frequencies of cantilever beam is presented . Hamilton's principle applied to the Lagrangian function is used to derive the equations of motion. In addition other factors affecting the vibration of beams are discussed. The variables of the hinged beam are:

1. Slenderness ratio
2. Shearing consideration

The numerical results for free vibration of beam are presented. These results are compared with the results obtained using MATLAB R2010a to plot the modal natural frequency of simply supported beam. The module frequencies can be highly useful for the vibration analysis and the resonance in a structure. So, the beam is taken and its module natural frequencies are computed.

CHAPTER~1

1. INTRODUCTION

Beam is a Horizontal or inclined structural member spanning a distance between one or more supports, and carrying vertical loads across (transverse to) its longitudinal axis, as a girder,purlin, or rafter. Three basic types of beams are:

(1) Simple span, supported at both ends

(2) Continuous, supported at more than two points

(3) Cantilever, supported at one end with the other end overhanging and free.

Generally there are two types of beams Euler-Bernoulli's beam and Timoshenko beam. By the classical theory of Euler-Bernoulli's beam it assumes that

1. Cross-sectional plane perpendicular to the axis of the beam remain plane after deformation.

2. The deformed cross-sectional plane is still perpendicular to the axis after deformation.

3. The classical theory of beam neglect the transverse shearing deformation, where the transverse shear is determined by the equation of equilibrium.

In Euler – Bernoulli beam theory, shear deformations and rotation effects are neglected, and plane sections remain plane and normal to the longitudinal axis. In the Timoshenko beam theory, plane sections still remain plane but are no longer normal to the longitudinal axis.

1.2 Objective and Scope of work

In this paper, we are using Finite Element Method to formulate the equations of motion of a homogeneous hinged-hinged type beam. The natural frequency of the homogeneous beam will be found out at different variables of beam using MATLAB R2010 . The results will be compared with the results found by finite element method. Using these results, frequency and beam variables will be correlated.

CHAPTER~2

2. LITERATURE SURVEY

An exact formulation of the beam problem was first investigated in terms of general elasticity equations by Pochhammer (1876) and Chree (1889) . They derived the equations that describe a vibrating solid cylinder. However, it is not practical to solve the full problem because it yields more information than usually needed in applications. Therefore, approximate solutions for transverse displacement are sufficient. The beam theories under consideration all yield the transverse displacement as a solution.
It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam. The Euler Bernoulli model includes the strain energy due to the bending and the kinetic energy due to the lateral displacement. The Euler Bernoulli model dates back to the 18th century. Jacob Bernoulli (1654-1705) first discovered that the curvature of an elastic beam at any point is proportional to the bending moment at that point. Daniel Bernoulli (1700-1782), nephew of Jacob, was the first one who formulated the differential equation of motion of a vibrating beam. Later, Jacob Bernoulli's theory was accepted by Leonhard Euler (1707-1783) in his investigation of the shape of elastic beams under various loading conditions. Many advances on the elastic curves were made by Euler . The Euler-Bernoulli beam theory, sometimes called the classical beam theory, Euler beam theory, Bernoulli beam theory, or Bernoulli and Euler beam theory, is the most commonly used because it is simple and provides reasonable engineering approximations for many problems. However, the Euler Bernoulli model tends to slightly overestimate the natural frequencies. This problem is exacerbated for the natural frequencies of the higher modes. Also, the prediction is better for slender beams than non-slender beams.

Timoshenko (1921, 1922) proposed a beam theory which adds the effect of shear as well as the effect of rotation to the Euler-Bernoulli beam. The Timoshenko model is a major improvement for non-slender beams and for high-frequency responses where shear or rotary effects are not negligible. Following Timoshenko, several authors have obtained the frequency equations and the mode shapes for various boundary conditions. Some are Kruszewski (1949) , Traill-Nash and Collar (1953) , Dolph (1954) , and Huang (1961) .

The finite element method originated from the need of solving complex elasticity and structural analysis problem in civil and aeronautical engineering. Its development could be traced back to the work by Alexander Hrennikoff (1941) and Richard Courant (1942). While the approach used by these pioneers are different, they all share one essential characteristic: mesh discretization of a continuous domain into a set of discrete subdomains, usually called elements. Starting in 1947, Olgierd Zienkiewicz from Imperial College gathered those methods together into what is called the Finite Element Method, building the pioneering mathematical formalism of the method.

Hrennikofs work discretizes the domain by using a lattice analogy, while Courant's approach divides the domain into finite triangular subregions to solve second order elliptic partial differential equations (PDEs) that arise from the problem of torsion of a cylinder. Courant's contribution was evolutionary, drawing on a large body of earlier results for PDEs developed by Rayleigh, Ritz, andGalerkin.

Development of the finite element method began in the middle to late 1950s for airframe and structural analysis and gathered momentum at the University of Stuttgartthrough the work of John Argyris and at Berkeley through the work of Ray W. Clough in the 1960's for use in civil engineering. By late 1950s, the key concepts of stiffness matrix and element assembly existed essentially in the form used today. NASA issued a request for proposals for the development of the finite element software NASTRAN in 1965. The method was again provided with a rigorous mathematical foundation in 1973 with the publication of Strang and Fix *An Analysis of The Finite Element Method*, and has since been generalized into a branch of applied mathematics for numerical modelling of physical systems in a wide variety of engineering disciplines, e.g., electromagnetism and fluid dynamics.

CHAPTER~3

3. Numerical modeling and formulation

1. Formulation:

EULER BERNOULLI BEAM:

For stiffness matrix:

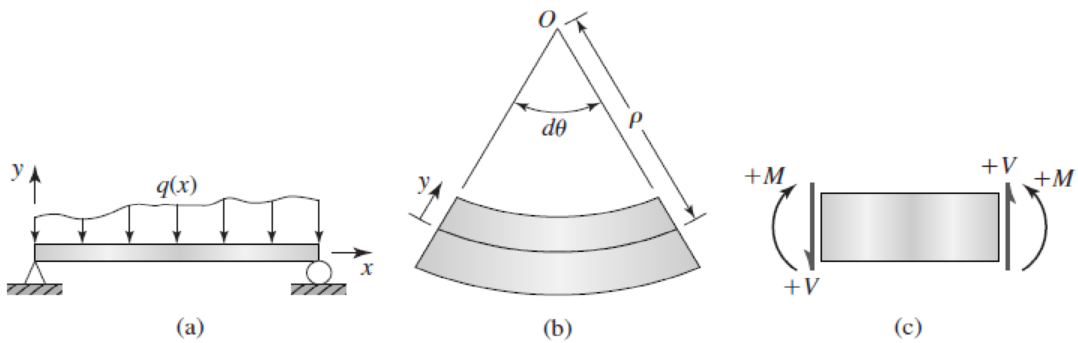


Fig: (a) Simply supported beam subjected to arbitrary (negative) distributed load.(b) Deflected beam element. (c) Sign convention for shear force and bending moment.

The bending strain is:

$$\varepsilon_x = \frac{ds - dx}{dx} = \frac{(\rho - y)\,d\theta - \rho\,d\theta}{\rho\,d\theta} = -\frac{y}{\rho}$$

The radius of curvature of a given curve is:

$$\rho = \frac{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}}{\frac{d^2v}{dx^2}}$$

the term below can be neglected:

$$\left(\frac{dv}{dx}\right)^2$$

$$\varepsilon_x = -y\frac{d^2v}{dx^2}$$

therefore :

$$U_e = \frac{1}{2}\int_V \sigma_x \varepsilon_x \, dV$$

is the total strain energy...

$$\varepsilon_x = -y\frac{d^2v}{dx^2} \qquad , \qquad \sigma_x = E\varepsilon_x = -Ey\frac{d^2v}{dx^2}$$

$$U_e = \frac{E}{2}\int_0^L \left(\frac{d^2v}{dx^2}\right)^2 \left(\int_A y^2 \, dA\right) dx$$

$$I = \left(\int_A y^2 \, dA\right)$$

$$U_e = \frac{EI_z}{2}\int_0^L \left(\frac{d^2v}{dx^2}\right)^2 dx$$

Considering the given four boundary conditions and the one-dimensional nature of the given problem in terms of the independent variable, we assume the displacement function in the form:

$$v(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$v(x = 0) = v_1 = a_0$$

$$v(x = L) = v_2 = a_0 + a_1 L + a_2 L^2 + a_3 L^3$$

$$\left.\frac{dv}{dx}\right|_{x=0} = \theta_1 = a_1$$

$$\left.\frac{dv}{dx}\right|_{x=L} = \theta_2 = a_1 + 2a_2 L + 3a_3 L^2$$

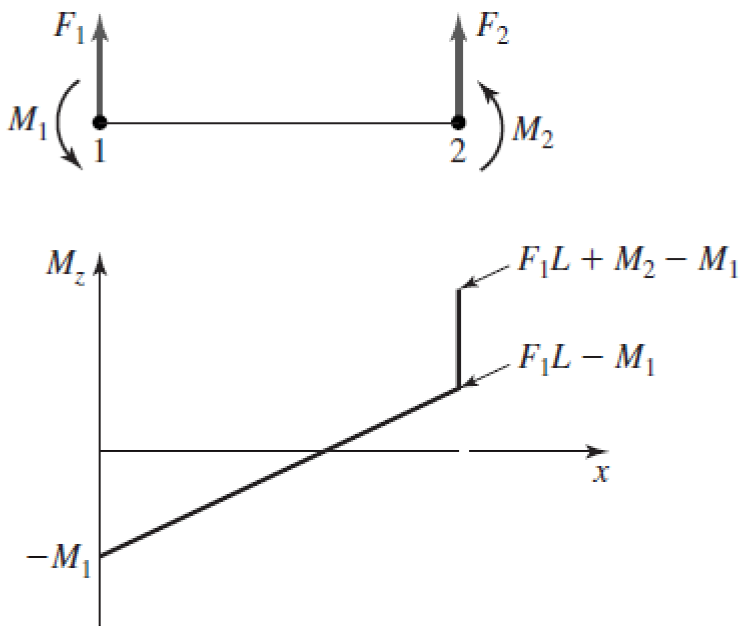


Fig: Bending moment diagram for a flexure element. Sign convention per the MOS theory.

$$v(x) = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)v_1 + \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)\theta_1 + \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right)v_2 + \left(\frac{x^3}{L^2} - \frac{x^2}{L}\right)\theta_2$$

Using the relation:

$$v(x) = [N_1 \quad N_2 \quad N_3 \quad N_4] \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = [N]\{\delta\}$$

where N_1, N_2, N_3, and N_4 are the shape functions that describe the distribution of displacement in terms of the nodal values in nodal displacement vector {δ}:

We get

$$U_e = \frac{EI_z}{2} \int_0^L \left(\frac{d^2 N_1}{dx^2} v_1 + \frac{d^2 N_2}{dx^2} \theta_1 + \frac{d^2 N_3}{dx^2} v_2 + \frac{d^2 N_4}{dx^2} \theta_2 \right)^2 dx$$

Applying the first theorem of Castigliano to the strain energy function with respect to nodal displacement v_1 gives the transverse force at node 1 as

$$\frac{\partial U_e}{\partial v_1} = F_1 = EI_z \int_0^L \left(\frac{d^2 N_1}{dx^2} v_1 + \frac{d^2 N_2}{dx^2} \theta_1 + \frac{d^2 N_3}{dx^2} v_2 + \frac{d^2 N_4}{dx^2} \theta_2 \right) \frac{d^2 N_1}{dx^2} dx$$

while application of the given theorem with respect to the rotational displacement results to moment as

$$\frac{\partial U_e}{\partial \theta_1} = M_1 = EI_z \int_0^L \left(\frac{d^2 N_1}{dx^2} v_1 + \frac{d^2 N_2}{dx^2} \theta_1 + \frac{d^2 N_3}{dx^2} v_2 + \frac{d^2 N_4}{dx^2} \theta_2 \right) \frac{d^2 N_2}{dx^2} dx$$

Similarly we obtain

$$\frac{\partial U_e}{\partial v_2} = F_2 \quad , \quad \frac{\partial U_e}{\partial \theta_2} = M_2$$

The above 4 equations can be represented in the form:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

By comparison of coefficients:

$$k_{mn} = k_{nm} = EI_z \int_0^L \frac{\mathrm{d}^2 N_m}{\mathrm{d}x^2} \frac{\mathrm{d}^2 N_n}{\mathrm{d}x^2}\,\mathrm{d}x \qquad m, n = 1, 4$$

Including dimensionless variable $\xi = x/L$

$$\int_0^L f(x)\,\mathrm{d}x = \int_0^1 f(\xi) L\,\mathrm{d}\xi$$

$$\frac{\mathrm{d}}{\mathrm{d}x} = \frac{1}{L}\frac{\mathrm{d}}{\mathrm{d}\xi}$$

The above equation becomes:

$$k_{mn} = k_{nm} = EI_z \int_0^L \frac{\mathrm{d}^2 N_m}{\mathrm{d}x^2} \frac{\mathrm{d}^2 N_n}{\mathrm{d}x^2}\,\mathrm{d}x = \frac{EI_z}{L^3} \int_0^1 \frac{\mathrm{d}^2 N_m}{\mathrm{d}\xi^2} \frac{\mathrm{d}^2 N_n}{\mathrm{d}\xi^2}\,\mathrm{d}\xi \qquad m, n = 1, 4$$

$$k_{mn} = k_{nm} = EI_z \int_0^L \frac{\mathrm{d}^2 N_m}{\mathrm{d}x^2} \frac{\mathrm{d}^2 N_n}{\mathrm{d}x^2}\,\mathrm{d}x \qquad m, n = 1, 4$$

The stiffness coefficients are:

$$k_{11} = \frac{EI_z}{L^3}\int_0^1 (12\xi - 6)^2\, d\xi = \frac{36EI_z}{L^3}\int_0^1 (4\xi^2 - 4\xi + 1)\, d\xi$$

$$= \frac{36EI_z}{L^3}\left(\frac{4}{3} - 2 + 1\right) = \frac{12EI_z}{L^3}$$

$$k_{12} = k_{21} = \frac{EI_z}{L^3}\int_0^1 (12\xi - 6)(6\xi - 4)L\, d\xi = \frac{6EI_z}{L^2}$$

$$k_{13} = k_{31} = \frac{EI_z}{L^3}\int_0^1 (12\xi - 6)(6 - 12\xi)\, d\xi = -\frac{12EI_z}{L^3}$$

$$k_{14} = k_{41} = \frac{EI_z}{L^3}\int_0^1 (12\xi - 6)(6\xi - 2)L\, d\xi = \frac{6EI_z}{L^2}$$

$$k_{22} = \frac{4EI_z}{L}$$

$$k_{23} = k_{32} = -\frac{6EI_z}{L^2}$$

$$k_{24} = k_{42} = \frac{2EI_z}{L}$$

$$k_{33} = \frac{12EI_z}{L^3}$$

$$k_{34} = k_{43} = -\frac{6EI_z}{L^3}$$

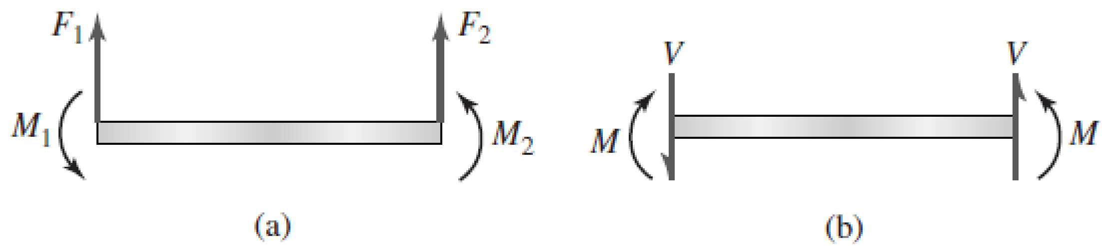
$$k_{44} = \frac{4EI_z}{L}$$

The complete stiffness value of flexure element is given as:

$$[k_e] = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Element load vector:

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} -V_1 \\ -M_1 \\ V_2 \\ M_2 \end{Bmatrix}$$



(a)nodal load positive convention(b)mechanics of solids positive convention theory

For mass matrix of the Euler-Bernoulli beam:

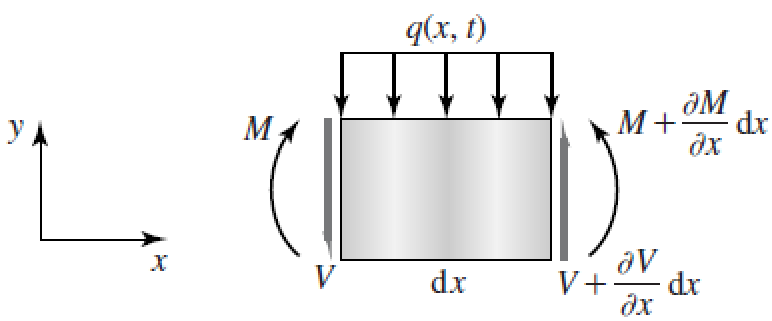


Fig:differential element of beam subjected to time dependent loading

From Newtons second law:

$$\sum F_y = ma_y \Rightarrow V + \frac{\partial V}{\partial x}\,\mathrm{d}x - V - q(x,t)\,\mathrm{d}x = (\rho A\;\mathrm{d}x)\frac{\partial^2 v}{\partial t^2}$$

We have:

$$-\frac{\partial^2 M}{\partial x^2} - q(x,t) = \rho A \frac{\partial^2 v}{\partial t^2}$$

On replacing the relation below in newtons second law

$$\frac{\partial M}{\partial x} = -V$$

Under the assumptions of constant elastic modulus E and moment of inertia I_z, the governing equation becomes:

$$\rho A \frac{\partial^2 v}{\partial t^2} + E I_z \frac{\partial^4 v}{\partial x^4} = -q(x,t)$$

On applying Galerkins method to the above equation,we have

$$\int_0^L N_i(x)\left(\rho A \frac{\partial^2 v}{\partial t^2} + E I_z \frac{\partial^4 v}{\partial x^4} + q\right) = 0 \qquad i = 1, 4$$

And thus we get:

$$\rho A \int_0^L [N]^T [N]\,\mathrm{d}x \begin{Bmatrix} \ddot{v}_1 \\ \ddot{\theta}_1 \\ \ddot{v}_2 \\ \ddot{\theta}_2 \end{Bmatrix} = \left[m^{(e)}\right] \begin{Bmatrix} \ddot{v}_1 \\ \ddot{\theta}_1 \\ \ddot{v}_2 \\ \ddot{\theta}_2 \end{Bmatrix}$$

The consistent mass matrix for a two-dimensional beam element is given by:

$$\left[m^{(e)}\right] = \rho A \int_0^L [N]^T [N]\,\mathrm{d}x$$

Substitution for the interpolation functions and performing the required integrations gives the mass matrix as

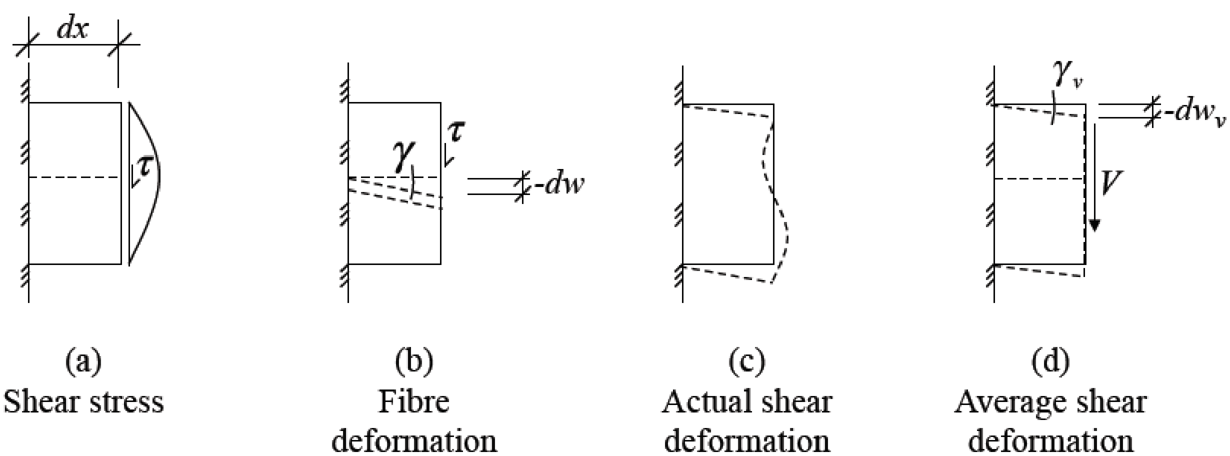
$$[m^{(e)}] = \frac{\rho A L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

Combining the mass matrix with previously obtained results for the stiffness matrix and force vector, the finite element equations of motion for a beam element are:

$$[m^{(e)}]\begin{Bmatrix} \ddot{v}_1 \\ \ddot{\theta}_1 \\ \ddot{v}_2 \\ \ddot{\theta}_2 \end{Bmatrix} + [k^{(e)}]\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = -\int_0^L [N]^T q(x,t)\, dx + \begin{Bmatrix} -V_1(t) \\ -M_1(t) \\ V_2(t) \\ M_2(t) \end{Bmatrix}$$

Timoshenko beam:

The shearing effect in Timoshenko beam element:



Consider an infinitesimal element of beam of length δx and flexural rigidity *EI*. The element is in static equilibrium under the forces shown in Figure

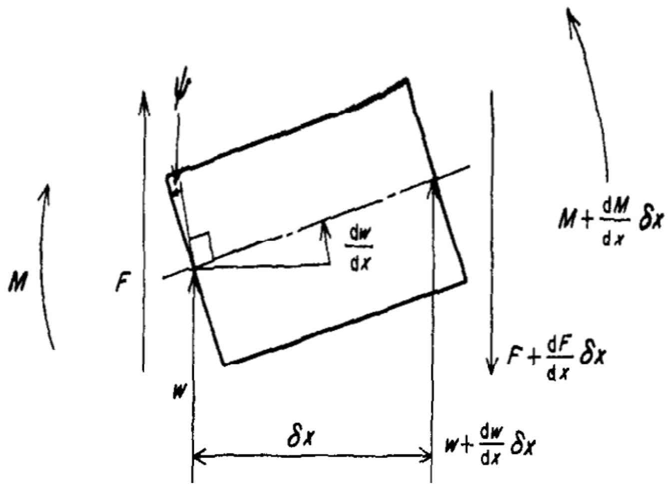


Fig: Forces and displacements on infinitesimal element of beam.

The shear angle, Ψ, is measured as positive in an anticlockwise direction from the normal to the midsurface to the outer face of the beam.

—

G-shear coeff.,k-shear modulus/shear factor

The static equilibrium relations are:

— ;—

The rotation of the cross section in an anticlockwise direction is:

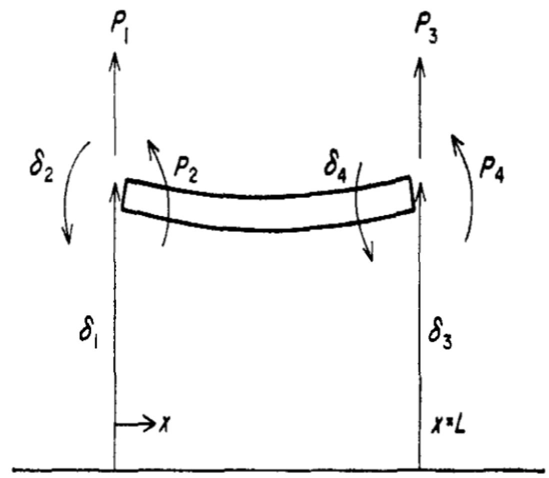
—

The stress-strain relation in bending is:

— — —

F=α1

M=α1x+α2



The rotations at the ends of the beam, δ2 and δ4 can be expressed as rotations of the cross section by using equation (4). The displacements δ1 to δ4 can be related to the constants α1 to α4 through:

for i=1,2,3,4

$$[\mathbf{X}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \beta & 0 & 1 & 0 \\ \frac{L^3}{6} & \frac{L^2}{2} & L & 1 \\ \frac{L^2}{2}+\beta & L & 1 & 0 \end{bmatrix},$$

$\{\boldsymbol{\delta}_i\} = \{w_1\,\theta_1\,w_2\,\theta_2\}_i^T$ and $\beta = EI/GKA$. Similarly the forces can be related to the constants by

{Pi}=[Y]{αi}

$$[\mathbf{Y}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ L & 1 & 0 & 0 \end{bmatrix}$$

and the elements of (Pi} are defined in Figure 2. Substituting for {αi} from equation (10) in equation (11) gives

=[S]{δi}

Where [S] is the stiffness matrix:

$$[\mathbf{S}] = \frac{EI}{L(L^2 + 12\beta)} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 + 12\beta & -6L & 2L^2 - 12\beta \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 - 12\beta & -6L & 4L^2 + 12\beta \end{bmatrix}.$$

The shear factor

Cross section	k'
Circle	$\frac{6(1+\nu)}{7+6\nu}$
Hollow circle with $m = r_{inner}/r_{outer}$	$\frac{6(1+\nu)(1+m^2)^2}{(7+6\nu)(1+m^2)^2 + (20+12\nu)m^2}$
Rectangle	$\frac{10(1+\nu)}{12+11\nu}$
Thin-walled round tube	$\frac{2(1+\nu)}{4+3\nu}$
Thin-walled square tube	$\frac{20(1+\nu)}{48+39\nu}$

$$\Phi_y = \frac{12EI_{yy}}{\kappa_z GAl^2}$$

Consider a typical two-node three-dimensional beam element of length l, where each node has six degrees of freedom. The nodal displacement vector $\{\mathbf{e}\}$ defined with respect to the element axes is denoted by

$$\{\mathbf{e}\}_{12\times 1}=[u_1 \quad v_1 \quad w_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad u_2 \quad v_2 \quad w_2 \quad \theta_{x2} \quad \theta_{y2} \quad \theta_{z2}]^{\mathrm{T}}, \tag{1}$$

The shape functions of the timoshenko beam are:

APPENDIX A

Shape function matrix for the three-dimensional Timoshenko beam element:

$$[\mathcal{N}]^{\mathrm{T}} = \begin{bmatrix} (1-\xi) & 0 & 0 \\ 6\bar{\Phi}_z(\xi-\xi^2)\eta & \bar{\Phi}_z(1-3\xi^2+2\xi^3+\Phi_z(1-\xi)) & 0 \\ 6\bar{\Phi}_y(\xi-\xi^2)\zeta & 0 & \bar{\Phi}_y(1-3\xi^2+2\xi^3+\Phi_y(1-\xi)) \\ 0 & -(1-\xi)l\zeta & (1-\xi)l\eta \\ l\bar{\Phi}_y(1-4\xi+3\xi^2+\Phi_y(1-\xi))\zeta & 0 & -l\bar{\Phi}_y\left(\xi-2\xi^2+\xi^3+\frac{\Phi_y}{2}(\xi-\xi^2)\right) \\ -l\bar{\Phi}_z(1-4\xi+3\xi^2+\Phi_z(1-\xi))\eta & l\bar{\Phi}_z\left(\xi-2\xi^2+\xi^3+\frac{\Phi_z}{2}(\xi-\xi^2)\right) & 0 \\ \xi & 0 & 0 \\ 6\bar{\Phi}_z(-\xi+\xi^2)\eta & \bar{\Phi}_z(3\xi^2-2\xi^3+\Phi_z\xi) & 0 \\ 6\bar{\Phi}_y(-\xi+\xi^2)\zeta & 0 & \bar{\Phi}_y(3\xi^2-2\xi^3+\Phi_y\xi) \\ 0 & -\xi l\zeta & \xi l\eta \\ l\bar{\Phi}_y(-2\xi+3\xi^2+\Phi_y\xi)\zeta & 0 & -l\bar{\Phi}_y\left(-\xi^2+\xi^3-\frac{\Phi_y}{2}(\xi-\xi^2)\right) \\ -l\bar{\Phi}_z(-2\xi+3\xi^2+\Phi_z\xi)\eta & l\bar{\Phi}_z\left(-\xi^2+\xi^3-\frac{\Phi_z}{2}(\xi-\xi^2)\right) & 0 \end{bmatrix},$$

where $\bar{\Phi}_y = 1/(1+\Phi_y)$ and $\bar{\Phi}_z = 1/(1+\Phi_z)$.

The mass matrix of timoshenko beam:

$$\bar{\mathrm{M}} = \frac{\rho AL}{840}\begin{bmatrix} 280 & 0 & 0 & 140 & 0 & 0 \\ & 312+588\Phi+280\Phi^2 & (44+77\Phi+35\Phi^2)L & 0 & 108+252\Phi+175\Phi^2 & -(26+63\Phi+35\Phi^2)L \\ & & (8+14\Phi+7\Phi^2)L^2 & 0 & (26+63\Phi+35\Phi^2)L & -(6+14\Phi+7\Phi^2)L^2 \\ & & & 280 & 0 & 0 \\ & \mathrm{SYM} & & & 312+588\Phi+280\Phi^2 & -(44+77\Phi+35\Phi^2)L \\ & & & & & (8+14\Phi+7\Phi^2)L^2 \end{bmatrix}$$

We have the boundary conditions:

$$\frac{\partial^2 v}{\partial x^2} = 0, \quad v = 0$$

for hinged end

CHAPTER ~4

4. Vibration analysis using MATLABR2010

The MATLAB code for the modal solution is:

```
function SSbeam(~)
% SSbeam.m Simply-supported or Pinned-pinned beam evaluations
% This script computes mode shapes and corresponding natural
% frequencies of the simply-supported beam by user specified mechanical
% properties and size of the beam.
% Prepare the following data:
%  - Material properties of the beam, i.e. density (Ro), Young's modulus (E)
%  - Specify a cross section of the beam, i.e. square,rectangular, circular
%  - Geometry parameters of the beam, i.e. Length, width, thickness
%  - How many natural frequencies and mode shapes to evaluate.


clear all;
clc;
close all;
display('What is the cross-section of the beam?')
disp('If circular cross-section, enter 1; If square, enter 2;');
disp('If rectangle enter 3');
disp('If your beam"s cross-section is not listed here, enter 4');
disp('To see example #2, enter 5');

CS=input(' Enter your choice:-    ');

if isempty(CS) || CS==0
    disp('Example #1. Rectangular cross-section Aluminum beam')
    disp('Length=0.321 [m], Width=0.05 [m], Thickness=0.006 [m];')
    disp('E=69.9*1e9 [Pa]; Ro=2770 [kg/m^3]')
    L=.321;
    W=.05;
    Th=.006;
    A=W*Th;
    Ix=(1/12)*W*Th^3;
    E=69.90e+9;
    Ro=2770;
elseif CS==1
    R=input('Enter Radius of the cross-section: ');
    L=input('Enter Length: ');
    Ix=(1/4)*pi*R^4;
    A=pi*R^2;
    disp('Material proprties of the beam');
    display('Do you know your beam"s material properties, viz. Young"s
modulus and density ?');
    YA=input('Enter 1, if you do;; enter 0, if you don"t:  ');
    if YA==1
```

```
    E=input('Enter Young's modulus in [Pa]:  ');
    Ro=input('Enter materials density in [kg/m^3]:  ');
    else
        display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3]');
        display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3]');
        display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3]');
        E=input('Enter Young's modulus in [Pa]: ');
        Ro=input('Enter materials density in [kg/m^3]:  ');
    end
elseif CS==2
    W=input('Enter Width of the cross-section: ');
    L=input('Enter Length:- ');
    Ix=(1/12)*W^4;
    A=W^2;
    disp('Material proprties of the beam');
    display('Do you know your beam's material properties, i.e. Young's
modulus and density ?');
    YA=input('Enter 1, if you do; enter 0, if you don't:  ');
    if YA==1
    E=input('Enter Young's modulus in [Pa] ');
    Ro=input('Enter the material density in [kg/m^3]  ');
    else
        display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3]')
        display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3]')
        display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3]')
        E=input('Enter Young's modulus in [Pa]:  ');
        Ro=input('Enter the material density in [kg/m^3]:  ');
    end
elseif CS==3
    W=input('Enter Width of the cross-section in [m]:  ');
    Th=input('Enter Thickness of the cross-section in [m]:  ');
    L=input('Enter Length in [m]:  ');
    Ix=(1/12)*W*Th^3;
    A=W*Th;
    disp('Material proprties of the beam')
    display('Do you know your beam's material properties, viz. Young's
modulus and density ?')
    YA=input('Enter 1, if you do; enter 0, if you don't:  ');
    if YA==1
    E=input('Enter Young's modulus in [Pa]:  ');
    Ro=input('Enter the material density in [kg/m^3]:  ');
    else
        display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3]  ')
        display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3]  ')
        display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3]  ')
        E=input('Enter Young's modulus in [Pa]:   ');
        Ro=input('Enter the material density in [kg/m^3]:   ');
    end
elseif CS==4
    display('Note: you need to compute Ix (area moment of inertia along x
axis) and X-sectional area')
    L=input('Enter Length in [m]:  ');
    Ix=('Enter Ix in [m^4]:  ');
    A=('Enter cross-x-sectional area in [m^2]:  ');
    disp('Material properties of the beam')
```

```
    display('Do you know your beam"s material properties, viz. Young"s
modulus and density ?');
    YA=input('Enter 1, if you do; enter 0, if you don"t:  ');
    if YA==1
    E=input('Enter Young"s modulus in [Pa]:  ');
    Ro=input('Enter material density in [kg/m^3]:  ');
    else
        display('Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3] ')
        display('Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3] ')
        display('Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3] ')
        E=input('Enter Young"s modulus in [Pa]:  ');
        Ro=input('Enter material density in [kg/m^3]:  ');
    end
elseif  CS==5
    display('Example #2')
    display('It is a rectangular X-section Aluminum beam ')
    display('Length=0.03; Width=0.005; Thickness=0.0005;')
    L=.03; W=.005; Th=.0005;
    A=W*Th;
    Ix=(1/12)*W*Th^3;
    E=70*1e9; Ro=2.7*1e3;
else
    F=warndlg('It is not clear what your choice of X-section of a beam is. e-
run so you can enter your beam"s data!!!','!! Warning !!');
    waitfor(F)
    display('Type in:>> SSbeam')
    pause(3)
    return
end

display('How many modes and mode shapes would you like to evaluate ?')
HMMS=input('Enter the number of modes and mode shapes to be computed: ');
if HMMS>=7
    disp('    ')
    warning('NOTE: Up to 6 mode shapes (plots) are displayed via the script.
Yet, using evaluated data (Xnx) of the script, more mode shapes can be
plotted');
    disp('   ')
end
    jj=1;
    while jj<=HMMS;
        betaNL(jj)=jj*pi;
        jj=jj+1;
    end
    %fprintf('betaNL value is %2.3f\n', betaNLall);
    betaN=(betaNL/L)';
    display('Mode shape corresponding nat. freq (fn) by Euler-Bernoulli
theory:')
    wn=zeros(1,length(betaN));
    fn=ones(1,length(wn));
    k=1;
    while k<=length(betaN);
        wn(k)=betaN(k)^2*sqrt((E*Ix)/(Ro*A));
        fn(k)=wn(k)/(2*pi);
```

```
        fprintf('Mode shape # %2f corresponds to nat. freq (fn): %3.3f\n', k,
fn(k) );
        k=k+1;
    end

    x=linspace(0, L, 720);
    xl=x./L;

    Xnx=zeros(length(betaN),length(x));
    for ii=1:length(betaN)
        for jj=1:length(x)
            Xnx(ii,jj)=sin((ii*pi*x(jj))/L);
        end
    end
    XnxMAX=max(abs(Xnx(1,1:end)));
    Xnx=Xnx./XnxMAX;
    % Plot mode shapes that are arbitrarily normalized to unity;
    display('NOTE: Upto 5 mode shapes are displayed via the script
options.');
    disp(' Yet, using evaluated data (Xnx) of the script, more mode shapes
can be plotted');
    MMS=HMMS;
    if MMS==1
        plot(xl,Xnx(1,:), 'b-')
        title('Mode shapes of the Pinned-pinned beam')
        legend('Mode #1', 0); xlabel('x/L'); ylabel('Mode shape X_n(x)')
        grid
        hold off
    elseif MMS==2
        plot(xl,Xnx(1,:), 'b-'); hold on
        plot(xl,Xnx(2,:), 'r-');grid
        title('Mode shapes of the Pinned-pinned beam')
        legend('Mode #1', 'Mode #2', 0)
        xlabel('x/L'); ylabel('Mode shape X_n(x)')
        hold off
    elseif MMS==3
        plot(xl,Xnx(1,:), 'b-'); hold on
        plot(xl,Xnx(2,:), 'r-')
        plot(xl,Xnx(3,:), 'm-');grid
        title('Mode shapes of the Pinned-pinned beam')
        legend('Mode #1', 'Mode #2', 'Mode #3', 0)
        xlabel('x/L'); ylabel('Mode shape X_n(x)')
        hold off
    elseif MMS==4
        plot(xl,Xnx(1,:), 'b-'); hold on
        plot(xl,Xnx(2,:), 'r-')
        plot(xl,Xnx(3,:), 'm-')
        plot(xl,Xnx(4,:), 'c-'); grid
        title('Mode shapes of the Pinned-pinned beam')
        legend('Mode #1', 'Mode #2', 'Mode #3', 'Mode #4', 0)
        xlabel('x/L'); ylabel('Mode shape X_n(x)')
        hold off
    elseif MMS==5
        plot(xl,Xnx(1,:), 'b-'); hold on
        plot(xl,Xnx(2,:), 'r-')
        plot(xl,Xnx(3,:), 'm-')
```

```
        plot(xl,Xnx(4,:), 'g-')
        plot(xl,Xnx(5,:), 'k-')
        grid
        title('Mode shapes of the Pinned-pinned beam');
        legend('Mode #1', 'Mode #2', 'Mode #3', 'Mode #4', 'Mode #5', 0);
        xlabel('x/L'); ylabel('Mode shape X_n(x)');
        hold off
    elseif MMS>=6
        plot(xl,Xnx(1,:), 'b-'); hold on
        plot(xl,Xnx(2,:), 'r-')
        plot(xl,Xnx(3,:), 'm-')
        plot(xl,Xnx(4,:), 'g-')
        plot(xl,Xnx(5,:), 'k-')
        plot(xl,Xnx(6,:), 'c-')
        grid
        title('Mode shapes of the Pinned-pinned beam')
        legend('Mode #1', 'Mode #2', 'Mode #3', 'Mode #4', 'Mode #5', 'Mode
#6', 0)
        xlabel('x/L'); ylabel('Mode shape X_n(x)')
        hold off
    end

    %% By Timoshenko beam theory, natural frequency of simply-supported beam
is
    % found to be:
    alphaSQ=E*Ix/(Ro*A);
    rSQ=Ix/A;
% Inertia (effects) is considred
disp('Nat. freq. by Timoshenko (Inertia alone considered):')
omegaNi=zeros(1,HMMS);
for ii=1:HMMS
    omegaNi(ii)=sqrt((alphaSQ*ii^4*pi^4)/(L^4*(1+(ii^2*pi^2*rSQ)/(L^2))));
    fprintf('mode # %2f  corresponds to nat. freq. (fn) %3.3f \n', ii,
omegaNi(ii)/(2*pi));
end

% Shear deformation is considred
EoverKG=[1, 2, 3];
for ii=1:HMMS
    for jj=1:3

omegaNs(ii,jj)=sqrt((alphaSQ*ii^4*pi^4)/(L^4*(1+(ii^2*pi^2*rSQ*EoverKG(jj))/(
L^2))));
    end
end
display('Nat. freq. by Timoshenko (Shear deformation alone considered)')
disp('for 3 values of E/kG = [1; 2; 3].')
fprintf(' %3.3f\t %3.3f \t%3.3f\t \n', omegaNs'/(2*pi))
disp('In order to use classical beam or Euler-Bernoulli beam calc"s. One
should ')
disp('pay attention to Length/width ratio that must be larger than 10')
FF=warndlg('pay attention to Length/width ratio that must be greater than 10
to use Euler-Bernoulli beam method!!!','!! Warning !!');
waitfor(FF)
end
```

CHAPTER~5

5. Results and Discussion

The mode shapes of a beam were calculated and the analysis was done using the finite element method by calculating the characterstic matrices(mass matix and stiffness matrix)of the given simply supported beam.The natural frequencies and the mode shape of the given hinged-hinged type beam were calculated using MATLAB . The natural frequencies can show the pattern of the resonance that a beam is going to follow and its effect on structures. The mode shapes of a structural steel beam with given slenderness ratio and circular cross section were calculated and the following result was obtained:

The output is:

What is the cross-section of the beam?

If circular cross-section, enter 1; If square, enter 2;

If rectangle, enter 3;

If your beam's cross-section is not listed here, enter 4

To see example #2, enter 5

Enter your choice: 1

Enter Radius of the cross-section: 25

Enter Length: 350

Material properties of the beam

Do you know your beam's material properties, i.e. Young's modulus and density ?

Enter 1, if you do; enter 0, if you don"t: 0

Steel: E=2.1e+11 [Pa]; Ro=7850 [Kg/m^3]

Copper: E=1.2e+11 [Pa]; Ro=8933 [Kg/m^3]

Aluminum: E=0.69e+11 [Pa]; Ro=2700 [Kg/m^3]

Enter Young"s modulus in [Pa]: 2.1e+11

Enter material density in [kg/m^3]: 7850

How many modes and mode shapes would you like to evaluate ?

Enter the number of modes and mode shapes to be computed: 6

Mode shape corresponding nat. freq (fn) by Euler-Bernoulli theory:

Mode shape # 1.000000 corresponds to nat. freq (fn): 0.829

Mode shape # 2.000000 corresponds to nat. freq (fn): 3.316

Mode shape # 3.000000 corresponds to nat. freq (fn): 7.461

Mode shape # 4.000000 corresponds to nat. freq (fn): 13.264

Mode shape # 5.000000 corresponds to nat. freq (fn): 20.726

Mode shape # 6.000000 corresponds to nat. freq (fn): 29.845

NOTE: Upto 5 mode shapes are displayed via the script options.

Yet, using evaluated data (Xnx) of the script, more mode shapes can be plotted

Nat. freq. by Timoshenko (Inertia alone considered):

mode # 1.000000 corresponds to nat. freq. (fn) 0.824

mode # 2.000000 corresponds to nat. freq. (fn) 3.236

mode # 3.000000 corresponds to nat. freq. (fn) 7.071

mode # 4.000000 corresponds to nat. freq. (fn) 12.102

mode # 5.000000 corresponds to nat. freq. (fn) 18.076

mode # 6.000000 corresponds to nat. freq. (fn) 24.758

Nat. freq. by Timoshenko (Shear deformation alone considered)

for 3 values of E/kG = [1; 2; 3].

0.824	0.819	0.814
3.236	3.161	3.091
7.071	6.737	6.446

12.102	11.199	10.473
18.076	16.236	14.864
24.758	21.615	19.429

In order to use classical beam or Euler-Bernoulli beam calculations. One should pay attention to Length/width ratio that must be larger than 10


File Edit View Insert Tools Desktop Window Help
Mode shapes of the Pinned-pinned beam
Mode shape X_n(x)
x/L
Mode #1
Mode #2
Mode #3
Mode #4
Mode #5
Mode #6

CHAPTER~6

6. Conclusion

In this report, we examined four approximate models for a transversely vibrating beam: the Euler-Bernoulli and Timoshenko models .The equation of motion and the boundary conditions were obtained and the frequency equations for four boundary conditions were obtained.

The circular cross section of a simply supported beam was analysed and the modal shapes and natural frequencies were calculated . The slight structural consideration will show that the amplitude of beam at resonance will be maximum and the problem of failure will arise.

So,in design considerations the beams taken should be such that there is no resonance for the stability of a structure.

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